6. INTEGRÁLSZÁMÍTÁS

6.1. $F: F(x) = x + \mathrm{C}, \quad \mathrm{C} \in \mathrm{R}$ (ez utóbbit a továbbiakban nem írjuk ki).

6.2. $F: F(x) = x^2 + C$.

6.3. $F: F(x) = \frac{2\sqrt{x^3}}{3} + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.4. $F: F(x) = \frac{x^4}{4} + \mathrm{C}$.

6.5. $F: F(x) = 2x^2 + 3x + \mathrm{C}$.

6.6. $F: F(x) = \frac{12\sqrt[4]{x^5}}{5} + 2x + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.7. $F: F(x) = -\frac{1}{2x^2} + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.8. $F: F(x) = \frac{3x\sqrt[3]{\mathrm{x}}}{4} + \mathrm{C}$.

6.9. $F: F(x) = \frac{3x^2\sqrt[3]{x^2}}{8} + \mathrm{C}$.

6.10. $F: F(x) = \frac{6x\sqrt[6]{x^5}}{11} + \mathrm{C}, \quad x \in \mathrm{R}_0^+$.

6.11. $F: F(x) = \frac{5x^4}{4} + x^3 - x^2 + 5x + \mathrm{C}$.

6.12. $F: F(x) = \frac{12x^3\sqrt[3]{x^2}}{11} - \frac{3x^2\sqrt[3]{x^2}}{2} + \frac{3x\sqrt[3]{x^2}}{5} + \mathrm{C}$.

6.13. $F: F(x) = \dfrac{3x^5}{5} - \dfrac{1}{2x^4} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.14. $F: F(x) = \dfrac{4}{5}x^2\ \sqrt{x} - x + 4\sqrt{x} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.15. $F: F(x) = \dfrac{x^3}{3} - 5\ln(-x) + \mathrm{C}, \quad x \in \mathrm{R}^-.$

6.16. $F: F(x) = x^3 - 2x^2 + 2x + \mathrm{C}.$

6.17. $F: F(x) = \dfrac{2}{3}\sqrt{x^3} + 2\ln x + \dfrac{3}{x} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.18. $F: F(x) = \dfrac{2}{9}\sqrt{x^9} - 4\sqrt{x^3} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.19. $F: F(x) = -\dfrac{(3-4x)^8}{32} + \mathrm{C}.$

6.20. $F: F(x) = \dfrac{\left(3x^2 + 2x + 1\right)^4}{8} + \mathrm{C}.$

6.21. $F: F(x) = -\dfrac{1}{5}\sqrt[4]{(1-4x)^5} + \mathrm{C}, \quad x \in \left]-\infty;\ \dfrac{1}{4}\right].$

6.22. $F: F(x) = \dfrac{\ln^3 x}{3} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.23. $F: F(x) = -\dfrac{2}{9}\sqrt{\left(4 - x^3\right)^3} + \mathrm{C}, \quad x \in \left]-\infty;\ \sqrt[3]{4}\right].$

6.24. $F: F(x) = -\sqrt{1-x^2} + \mathrm{C}, \quad x \in \left]-1;\ 1\right[.$

6.25. $F: F(x) = -\dfrac{1}{4\left(2 + 3x^2\right)^2} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.26. $F: F(x) = \frac{1}{3}\sqrt{\left(x^2+4\right)^3} + \mathrm{C}$.

6.27. $F: F(x) = \frac{2}{3}\ln(3x-1) + \mathrm{C}, \quad x \in \left]\frac{1}{3};\ +\infty\right[$.

6.28. $F: F(\mathrm{x}) = \ln(x^2 - 4x + 6) + \mathrm{C}$.

6.29. $F: F(x) = \frac{1}{18}\ln\left(4+9x^2\right) + \mathrm{C}$.

6.30. $F: F(x) = \ln\left(x^2 + x + 1\right) + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.31. $F: F(x) = \frac{1}{5}\ln(5x+3) + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.32. $F: F(x) = x - 5\ln(x+3) + \mathrm{C}, \quad x \in \left]\text{-}3;\ +\infty\right[$.

6.33. $F: F(x) = \ln\left(e^x + 1\right) + \mathrm{C}$.

6.34. $F: F(x) = \frac{5}{3}\ln\left(x^3+2\right) + \mathrm{C}, \quad x \in \mathrm{R}^+$.

6.35. $F: F(x) = \ln\left(e^x + x\right) + \mathrm{C}, \quad$.

6.36. $F: F(x) = \frac{e^{x^2}}{2} + \mathrm{C}$.

6.37. $F: F(x) = \frac{1}{3}e^{3x-5} + \mathrm{C}$.

6.38. $F: F(x) = \ln(\ln x) + \mathrm{C}, \quad x \in \left]1;\ +\infty\right[$.

6.39. $F: F(x) = \sqrt{1+e^{2x}} + \mathrm{C}$.

6.40. $F: F(x) = \frac{(2x+7)^7}{14} + \mathrm{C}$.

6.41. $F: F(x) = \frac{1}{5} e^{5x+1} + \mathrm{C}.$

6.42. $F: F(x) = \frac{\left(7x^2 + 2x - 3\right)^2}{4} + \mathrm{C}.$

6.43. $F: F(x) = \frac{5}{8} \ln^4 x + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.44. $F: F(x) = \frac{\left(\sqrt{x^3} - 2x\right)^2}{2} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.45. $F: F(x) = \frac{1}{6} \ln\left(3x^2 + 1\right) + \mathrm{C}.$

6.46. $F: F(x) = \ln\left(7x^6 + 5x^4 + 2x\right) + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.47. $F: F(x) = \frac{1}{4} \sqrt{4x^2 + 3} + \mathrm{C}.$

6.48. F: F(x) = $F(x) = \frac{8x \sqrt[8]{x^7}}{15} + \mathrm{C}, \quad x \in \mathrm{R}_0^+.$

6.49. $F: F(x) = \int x^{\frac{7}{10}} \, dx = \frac{10}{17} x^{\frac{17}{10}} + \mathrm{C}.$

6.50. $F: F(x) = \frac{x^4}{4} - \frac{10x\sqrt{x}}{3} + \frac{3}{\sqrt[3]{x^2}} - \frac{2}{3x^6} + \mathrm{C}.$

6.51. $F: F(x) = \frac{x^2}{4} + \frac{x}{2} - \frac{3}{2} \ln x - \frac{5}{2x} + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.52. $F: F(x) = 3e^{1+x} + \mathrm{C}.$

6.53. $F: F(x) = \frac{3 \cdot 2^x}{\ln 2} + 4 \ln x + \mathrm{C}, \quad x \in \mathrm{R}^+.$

6.54. $F: F(x) = \frac{3x^{\frac{5}{3}}}{5} + \frac{6x^{\frac{7}{6}}}{7} + \mathrm{C}, \quad x \in R^{+}.$

6.55. $F: F(x) = 2\sqrt{x} + \ln x + \mathrm{C}, \quad x \in R^{+}.$

6.56. $F: F(x) = -\frac{\left(1-e^{x}\right)^{4}}{4} + \mathrm{C}.$

6.57. $F: F(x) = \frac{2}{3\ln 2}\sqrt{\left(1+2^{x}\right)^{3}} + \mathrm{C}.$

6.58. $F: F(x) = \frac{1}{8}\int 8x\left(3+4x^{2}\right)^{\frac{1}{5}} dx = \frac{5}{48}\left(3+4x^{2}\right)^{\frac{6}{5}} + \mathrm{C}, \quad x \in R^{+}.$

6.59. $F: F(x) = \frac{3}{2}\int \frac{2x}{\sqrt{1+x^{2}}} dx = \frac{3}{2}\int 2x\left(1+x^{2}\right)^{-\frac{1}{2}} dx =$

$$= \frac{3}{2}\cdot 2\left(1+x^{2}\right)^{\frac{1}{2}} = 3\sqrt{1+x^{2}} + \mathrm{C}.$$

6.60. $F: F(x) = \frac{1}{2}\int 2x\left(x^{2}+6\right)^{-\frac{1}{2}} dx = \sqrt{x^{2}+6} + \mathrm{C}.$

6.61. $F: F(x) = e^{2}\int e^{5x} dx = \frac{e^{2}}{5}\int 5\cdot e^{5x} dx = \frac{e^{2}}{5}\cdot e^{5x} + \mathrm{C}.$

6.62. $F: F(x) = \frac{3}{2}\left(1+e^{x}\right)^{\frac{2}{3}} + \mathrm{C}.$

6.63. $F: F(x) = -\frac{1}{5}\int -5\cdot e^{-5x} dx + \int 3dx = -\frac{1}{5}\cdot e^{-5x} + 3x + \mathrm{C}.$

6.64. $F: F(x) = \int\left(e^{x} - \frac{1}{x}\right) dx = e^{x} - \ln|x| + \mathrm{C}, \quad x \in R\setminus\{0\}.$

6.65. $F: F(x) = \frac{3}{2}\sqrt[3]{\ln^{2} x} + \mathrm{C}, \quad x \in]1;\ \infty[.$

6.66. $F\colon F(x)=\dfrac{\ln^5 x}{5}+\mathrm{C},\quad x\in\mathrm{R}^+.$

6.67. $F\colon F(x)=\int\left(\dfrac{1}{x^2}-\dfrac{2}{x}+1\right)dx=-\dfrac{1}{x}-2\ln|x|+x+\mathrm{C},\quad x\in\mathrm{R}\setminus\{0\}.$

6.68. $F\colon F(x)=\dfrac{1}{2}\int\dfrac{2x+2}{x^2+2x-1}\,dx=\dfrac{1}{2}\ln|x^2+2x-1|+\mathrm{C},\quad x\in R^+.$

6.69. $F\colon F(x)=\int\dfrac{e^x+1}{e^x+e^{-x}+2}\,dx=\int\dfrac{e^x\left(e^x+1\right)}{\left(e^x+1\right)^2}\,dx=\ln\left(e^x+1\right)+C.$

6.70. $F\colon F(x)=5\int(x-2)^{-4}\,dx=\dfrac{-5}{3}\cdot(x-2)^{-3}+\mathrm{C},\quad x\in\mathrm{R}\setminus\{2\}.$

6.71. $F\colon F(x)=\dfrac{2x^3}{3}-2x^2+x+2.$

6.72. $F\colon F(x)=5\ln|x|+\mathrm{C}.$

$5\ln|-e|+\mathrm{C}=-8,$

$5+\mathrm{C}=-8,$

$\mathrm{C}=-13.$

$F(x)=5\ln|x|-13.$

6.73. $F\colon F(x)=2x^2-2x-12.$

6.74. $\int x\cdot 3^x\,dx=\dfrac{x\cdot 3^x}{\ln 3}-\int\dfrac{3^x}{\ln 3}\,dx=\dfrac{x\cdot 3^x}{\ln 3}-\dfrac{1}{\ln 3}\cdot\dfrac{3^x}{\ln 3}=\dfrac{3^x}{\ln 3}\left(x-\dfrac{1}{\ln 3}\right)+\mathrm{C},$

mivel az eredeti integrál mögötti kifejezésre az $f'(x)=1$ és $g(x)=\dfrac{3^x}{\ln 3}$ helyettesítéseket célszerű bezetni.

6.75. $\int 1\cdot\ln x\,dx=x\cdot\ln x-\int x\cdot\dfrac{1}{x}\,dx=x\cdot\ln x-x=x(\ln x-1)+\mathrm{C},$

mivel az eredeti integrál mögötti kifejezésre az $f'(x)=1$ és $g(x)=\ln x$ helyettesítéseket célszerű bezetni.

6.76. $\int \left(x^3-5x+1\right)\ln x dx = \left(\frac{x^4}{4}-\frac{5x^2}{2}+x\right)\ln x - \int\left(\frac{x^3}{4}-\frac{5x}{2}+1\right)dx =$

$= \left(\frac{x^4}{4}-\frac{5x^2}{2}+x\right)\ln x - \frac{x^4}{16}+\frac{5x^2}{4}-x+\mathrm{C},$

mivel az eredeti integrál mögötti kifejezésre az $f'(x)=x^3-5x+1$ és $g(x)=\ln x$ helyettesítéseket célszerű bezetni.

6.77. $\int 1\cdot\ln^2 x dx = x\cdot\ln^2 x - \int\frac{x\cdot 2\ln x}{x} = x\cdot\ln^2 x - \int 2\ln x dx =,$

$= x\cdot\ln^2 x - 2(x\cdot\ln x-1)+\mathrm{C}$, mivel az eredeti integrál mögötti kifejezésre az $f'(x)=1$ és $g(x)=\ln^2 x$ helyettesítéseket célszerű bezetni.

6.78. $\frac{1}{2}\int x^2\cdot\frac{2x}{\sqrt{1+x^2}}dx = \frac{1}{2}\left(x^2\cdot 2\sqrt{1+x^2}-\int 4x\sqrt{1+x^2}dx\right) =$

$= x^2\sqrt{1+x^2}-\frac{2}{3}\sqrt{\left(1+x^2\right)^3}+\mathrm{C}.$

6.79. $f'(x)=x^3$, illetve $g(x)=\ln x$ helyettestéseket bevezetve:

$\int_1^e x^3\ln x dx = \frac{x^4\cdot\ln x}{4}-\int_1^e\frac{x^3}{4}dx = \left[\frac{x^4\ln x}{4}-\frac{x^4}{16}\right]_1^e = \left[\frac{4x^4\ln x - x^4}{16}\right]_1^e =$

$= \frac{1}{16}\left(3e^4+1\right).$

6.80. $\left[\frac{x^3}{3}\right]_1^3 = \frac{26}{3}.$

6.81. $\left[\frac{e^{2x}}{2}\right]_0^2 = \frac{e^4}{2}-\frac{1}{2} = 26{,}8.$

6.82. $\left[\frac{x^2}{4}-\frac{x}{4}\right]_1^6=\frac{15}{2}.$

6.83. $\left[\ln(-x)\right]_{-e}^{-1}=-1.$

6.84. $\left[-\frac{1}{3}\sqrt{\left(1-x^2\right)^3}\right]_0^{0,5}=-\frac{1}{3}\sqrt{0,75^3}+\frac{1}{3}=0,1168.$

6.85. $-\left[\frac{1}{6(2x+1)^3}\right]_2^3=0,00085.$

6.86. $\int_1^2\frac{x+1}{2x-7}\,dx=\frac{1}{2}\int_1^2\frac{2x-7+9}{2x-7}\,dx=\frac{1}{2}\int_1^2 1+\frac{9}{2x-7}\,dx=$

$=\frac{1}{2}\int_1^2 1dx+\frac{9}{4}\int_1^2\frac{2}{2x-7}\,dx=\left[\frac{1}{2}x+\frac{9}{4}\ln|2x-7|\right]_1^2=$

$=1+\frac{9}{4}\cdot\ln|-3|-\frac{1}{2}-\frac{9}{4}\cdot\ln|-5|\approx 0,35.$

6.87. $-\frac{3}{2}\int -2(3x-2)e^{-3x^2+4x}\,dx=\left[-\frac{3}{2}\cdot e^{-3x^2+4x}\right]_0^1=$

$=-\frac{3}{2}e+\frac{3}{2}=\frac{3}{2}(1-e)\approx -2,57.$

6.88. a.) $5\int\left(6x^2+1\right)\left(4x^3+2x\right)^{\frac{2}{3}}dx=\frac{5}{2}\int 2\left(6x^2+1\right)\left(4x^3+2x\right)^{\frac{2}{3}}dx=$

$=\frac{5}{2}\cdot\frac{3}{5}\left(4x^3+2x\right)^{\frac{5}{3}}=\frac{3}{2}\left(4x^3+2x\right)^{\frac{5}{3}}+C.$

b.) $\left[\frac{3}{2}\left(4x^3+2x\right)^{\frac{5}{3}}\right]_0^1=\frac{3}{2}\cdot\sqrt[3]{6^5}=\frac{3}{2}\sqrt[3]{7776}\approx 29,7.$

6.89. $(x-4)^2+1=-x^2+14x-39$,

$x^2-11x+28=0 \Rightarrow x_1=4,\ x_2=7.$

$$T=\int_4^7\left(-f(x)+g(x)\right)dx=2\int_4^7\left(-x^2+11x-28\right)dx=$$

$$=2\left[\frac{-x^3}{3}+\frac{11x^2}{2}-28x\right]_4^7=9.$$

6.90. $\left[\frac{1}{2}\ln\left(x^2+6x+10\right)\right]_0^1 = 0{,}2653.$

6.91. $\left[\frac{1}{2}e^{x^2}\right]_0^2 = 26{,}8.$

6.92. $\left[\frac{x^3}{3}+x^2+5x\right]_0^1 = \frac{19}{3}.$

6.93. $\left[\frac{\left(5x^3-2x^2+1\right)^2}{2}\right]_2^3 = 6417{,}5.$

6.94. $\left[\frac{1}{3}\ln\left(8x^3+3x\right)\right]_1^4 = 1{,}2879.$

6.95. $\left[-\frac{1}{3}\ e^{2-3x}\right]_{-2}^0 = 991{,}19.$

6.96. $\int_0^1\left(x^{\frac{3}{2}}-3e\cdot e^x\right)dx=\left[\frac{2x^{\frac{5}{2}}}{5}-3e\cdot e^x\right]_0^1=\frac{2}{5}-3e^2+3e\approx -13{,}61.$

6.97. $\frac{1}{10}\left(27\sqrt[3]{4}-21\right)\approx 2{,}19.$

6.98. $\left[\frac{1}{3}\ln\left(e^{3x}+1\right)\right]_1^2=0{,}985.$

6.99. $\frac{3}{4}\int_1^e \frac{4\left(x^3+2\right)}{x^4+8x}\,dx=\frac{3}{4}\left[\ln\left|x^4+8x\right|\right]_1^e=$

$=\frac{3}{4}\left[\ln\left(e^4+8e\right)-\ln 9\right]\approx 1{,}60.$

6.100. $\left[2\sqrt{1+\ln x}\right]_1^e=2\sqrt{1+1}-2=2\sqrt{2}-2=2\left(\sqrt{2}-1\right)\approx 0{,}83.$

6.101. $\frac{5}{3}\int_1^e \frac{3\left(x^2+2\right)}{x^3+6x}\,dx=\frac{5}{3}\left[\ln\left|x^3+6x\right|\right]_1^e=$

$=\frac{5}{3}\left(\ln\left(e^3+6e\right)-\ln 7\right)\approx 2{,}75.$

6.102. $\left[-\frac{1}{1+x^2}\right]_{-1}^0=-\frac{1}{2}.$

6.103. $\lim\limits_{b\to\infty}\int_1^b x^{-2}dx=\lim\limits_{b\to\infty}\left[-\frac{1}{x}\right]_1^b=1.$

6.104. $\int_0^1 \frac{1}{x^2}\,dx=\lim\limits_{a\to 0}\int_a^1 x^{-2}dx=\lim\limits_{a\to 0}\left[-\frac{1}{x}\right]_a^1=-1+\lim\limits_{a\to 0+0}\frac{1}{a}=\infty.$

6.105. $\int_1^\infty \frac{1}{x^4}\,dx=\lim\limits_{b\to\infty}\int_1^b x^{-4}dx=-\frac{1}{3}\lim\limits_{b\to\infty}\left[x^{-3}\right]_1^b=$

$=-\frac{1}{3}\lim\limits_{b\to\infty}\left[\frac{1}{b^3}-1\right]=\frac{1}{3}.$

6.106. 1.

6.107. $\frac{3}{5}\sqrt[3]{25} \approx 1{,}75.$

6.108. $e^{-1} \approx 0{,}37.$

6.109. $\lim\limits_{a\to 0}\lim\limits_{b\to\infty} -2\int\limits_a^b \left(-\frac{1}{2}\right)x^{-\frac{1}{2}}\cdot e^{-\sqrt{x}}dx = \lim\limits_{a\to 0}\lim\limits_{b\to\infty}(-2)\left[e^{-\sqrt{x}}\right]_a^b = 2.$

6.110. $\frac{1}{4}.$

6.111. $e^{-3} \approx 0{,}05.$

6.112. $\infty.$

6.113. $\frac{1}{2}.$

6.114. $\int\limits_{-1}^{0}\frac{1}{\sqrt[3]{x}}dx = \lim\limits_{a\to 0-0}\int\limits_{-1}^{a}x^{-\frac{1}{3}}dx = \frac{3}{2}\lim\limits_{a\to 0-0}\left[\sqrt[3]{x^2}\right]_{-1}^{a} =$

$= \frac{3}{2}\lim\limits_{a\to 0-0}\left[\sqrt[3]{a^2}-1\right] = -\frac{3}{2}.$

6.115. $\infty.$

6.116. 2.

6.117. $\infty.$

6.118. $-\frac{3}{2}.$

6.119. $\int\limits_0^2\frac{1}{\sqrt[3]{(x-1)^2}}dx = \lim\limits_{a\to 1-0}\int\limits_0^a\frac{dx}{\sqrt[3]{(x-1)^2}} + \lim\limits_{b\to 1+0}\int\limits_b^2\frac{dx}{\sqrt[3]{(x-1)^2}} = 6.$

6.120. $\int\limits_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -2 \lim\limits_{b\to\infty} \lim\limits_{a\to 0+0} \int\limits_a^b \left(-\frac{1}{2}\cdot\frac{1}{\sqrt{x}}\right) e^{-\sqrt{x}}\,dx =$

$= 2 \lim\limits_{b\to\infty} \lim\limits_{a\to 0+0} \left[e^{-\sqrt{x}} \right]_a^b = -2 \lim\limits_{b\to\infty} \lim\limits_{a\to 0+0} \left[\frac{1}{e^{\sqrt{b}}} - \frac{1}{e^{\sqrt{a}}} \right] = 2.$

6.121. a.) $f'(x) = \frac{1}{x^2}$, illetve $g(x) = \ln x$ helyettesítéseket alkalmazva:

$\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + \mathrm{C}.$

b.) $-\frac{\ln 1}{1} - \frac{1}{1} + \mathrm{C} = 10 \quad \Rightarrow \quad \mathrm{C} = 11,$

$F(x) = -\frac{\ln x}{x} - \frac{1}{x} + 11.$

6.122. a.) $f'(x) = e^{x+4}$, illetve $g(x) = 3x+5$ helyettesítéseket alkalmazva:

$\int (3x+5)e^{x+4}\,dx = (3x+5)e^{x+4} - \int 3\cdot e^{x+4}\,dx = (3x+5)e^{x+4} - 3e^{x+4} =$

$= e^{x+4}(3x+5-3) = e^{x+4}(3x+2) + \mathrm{C}.$

b.) $e^4 \cdot 2 + \mathrm{C} = 5 \quad \Rightarrow \quad \mathrm{C} = 5 - 2e^4,$

$F(x) = e^{x+4}(3x+2) + 5 - 2e^4.$

6.123. a.) $\int x\cdot e^{2x}\,dx = x\cdot\frac{e^{2x}}{2} - \int \frac{1}{2} e^{2x}\,dx = e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) + \mathrm{C}.$

b.) $\left[e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right) \right]_0^1 = e^2\left(\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{4} = \frac{1}{4}\left(e^2 + 1\right) \approx 2{,}08.$

6.124. $f'(x) = 1$, illetve $g(x) = -e^{-x}$ helyettesítéseket alkalmazva:

$\int\limits_0^1 x\cdot e^{-x}\,dx = \left[-x\cdot e^{-x} \right]_0^1 + \int\limits_0^1 e^{-x}\,dx = \left[-x\cdot e^{-x} - e^{-x} \right]_0^1 =$

$= -e^{-1} - e^{-1} + 1 = 1 - 2e^{-1}.$

6.125. $\left[\frac{x^3}{3} \right]_1^a = 2, \qquad \frac{a^3}{3} - \frac{1}{3} = 2, \qquad a = \sqrt[3]{7} = 1{,}913.$

6.126. $\left[\frac{x^2}{2}\right]_1^a = \left[\frac{x^2}{2}\right]_0^2, \qquad \frac{a^2}{2} - \frac{1}{2} = 2, \qquad a = \sqrt{5} = 2{,}236.$

6.127. $\left[\frac{x^3}{3}\right]_{-a}^a = 1, \qquad \frac{a^3}{3} + \frac{a^3}{3} = 1, \qquad a = \sqrt[3]{1{,}5} = 1{,}145.$

6.128. $\left[-\frac{1}{x}\right]_a^2 = 1, \qquad -\frac{1}{2} + \frac{1}{a} = 1, \qquad a = \frac{2}{3}.$

6.129. $\left[\ln(-x)\right]_a^{-1} = -1, \qquad \ln 1 - \ln(-a) = -1, \qquad a = -e.$

6.130. $\left[2x\right]_{-3}^1 + \left[\frac{2x^3}{3} + x\right]_1^2 + \left[4x\right]_2^4 = 21\frac{2}{3}.$

6.131. $\left[x\right]_0^1 + \left[\frac{x^4}{4} + x\right]_1^3 + \left[10x\right]_3^{10} = 93.$

6.132. $\int_{-1}^{2}\left(x + 2 - x^2\right)dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^2 = 4{,}5.$

6.133. $\int_0^1\left(\sqrt{x} - x^2\right)dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 = \frac{1}{3}.$

6.134. Metszéspontok: $\frac{4}{3x} = \frac{13}{3} - x \;\Rightarrow\; x_1 = \frac{1}{3},\; x_2 = 4.$

$$\mathrm{T} = \int_{\frac{1}{3}}^{4}\left(g(x) - f(x)\right)dx = \left[\frac{13x}{3} - \frac{x^2}{2} - \frac{4}{3}\ln x\right]_{\frac{1}{3}}^4 \approx 4{,}63.$$

6.135. Metszéspontok: $\frac{5}{2x}=\frac{11}{2}-x \;\Rightarrow\; x_1=\frac{1}{2},\;\; x_2=5.$

$$\mathrm{T}=\int_{\frac{1}{2}}^{5}\big(g(x)-f(x)\big)dx=\left[\frac{11x}{2}-\frac{x^2}{2}-\frac{5}{2}\ln|x|\right]_{\frac{1}{2}}^{5}=$$

$$=\frac{55}{2}-\frac{25}{2}-\frac{5}{2}\ln 5-\frac{11}{4}+\frac{1}{8}+\frac{5}{2}\ln\frac{1}{2}\approx 6{,}615.$$

6.136. Metszéspontok: $x+1=-x^2+3 \;\Rightarrow\; x_1=1,\;\; x_2=-2.$

$$\mathrm{T}=\int_{-2}^{1}\big(g(x)-f(x)\big)dx=\left[\frac{-x^3}{3}+2x-\frac{x^2}{2}\right]_{-2}^{1}=4{,}5.$$

6.137. Metszéspontok: $9-x^2=x+3,$

$$x^2+x-6=0,$$

$$x_{1,2}=\frac{-1\pm\sqrt{1+24}}{2}=\begin{cases}x_1=2,\\ x_2=-3,\end{cases}$$

$$\int_{-3}^{2}\Big(9-x^2-x-3\Big)dx=\int_{-3}^{2}\Big(6-x^2-x\Big)dx=\left[6x-\frac{x^3}{3}-\frac{x^2}{2}\right]_{-3}^{2}=$$

$$=12-\frac{8}{3}-2+18-9+\frac{9}{2}\;=\;23{,}5-\frac{8}{3}=20{,}83.$$

6.138. $$\int_{-1}^{1}\frac{x^2}{\sqrt{3+x^3}}\,dx=\frac{1}{3}\int_{-1}^{1}3x^2\left(x^3+3\right)^{-\frac{1}{2}}\,dx=\frac{2}{3}\left[\sqrt{x^3+3}\right]_{-1}^{1}=$$

$$=\frac{4}{3}-\frac{2\sqrt{2}}{3}\approx 0{,}39.$$

6.139. $\frac{1}{2x-3}>0$ a $\big[6;\;10\big]$ intervallumon.

$$\int_{6}^{10}\frac{1}{2x-3}\,dx=\left[\frac{1}{2}\ln\big(2x-3\big)\right]_{6}^{10}=\frac{1}{2}\big(\ln 17-\ln 9\big)=0{,}318.$$

6.140. Metszéspontok: $x_1 = 5, \quad x_2 = \frac{3}{2}$.

$$\int_{\frac{3}{2}}^{5}\left(-\frac{2}{x-1}-x+5{,}5\right)dx = \left[-2\ln|x-1|-\frac{x^2}{2}+5{,}5x\right]_{\frac{3}{2}}^{5} = 3{,}72.$$

6.141. Metszéspontok: $x_1 = \frac{-5}{2}, \quad x_2 = 1.$

$$\int_{\frac{-5}{2}}^{1}\left(-x+\frac{3}{2}-\frac{2}{x+3}\right)dx = \left[\frac{-x^2}{2}+\frac{3x}{2}-2\ln|x+3|\right]_{\frac{-5}{2}}^{1} = 3{,}72.$$

6.142. $T = \int_0^1\left(x^2-4x+3\right)dx + \left|\int_1^3\left(x^2-4x+3\right)dx\right| =$

$$= \left[\frac{x^3}{3}-2x^2+3x\right]_0^1 + \left|\left[\frac{x^3}{3}-2x^2+3x\right]_1^3\right| = \frac{8}{3}.$$