3. DIFFERENCIÁLHÁNYADOS ÉS DERIVÁLT

3.1. x a $\dfrac{(3x+1)-7}{x-2}=\dfrac{3(x-2)}{x-2}=3, \quad x\in\mathrm{R}\setminus\{2\}.$

3.2. x a $\dfrac{\sqrt[3]{x}-1}{x-1}=\dfrac{\sqrt[3]{x}-1}{\left(\sqrt[3]{x}-1\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=\dfrac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}, \quad x\in\mathrm{R}\setminus\{1\}.$

3.3. x a $\dfrac{\left(2x^2-3\right)+1}{x-1}=2\mathrm{x}+2, \quad x\in\mathrm{R}\setminus\{1\}$

3.4. x a $\dfrac{f(x)-f\left(e^2\right)}{x-e^2}=\dfrac{\ln x-\ln e^2}{x-e^2}=\dfrac{\ln x-2}{x-e^2}, \quad x\in\mathrm{R}^+\setminus\left\{e^2\right\}.$

3.5. $f'_-(2) = \lim\limits_{2-0}\dfrac{x^2+1-5}{x-2}=4,$

$f'_+(2) = \lim\limits_{2+0}\dfrac{2x+1-5}{x-2}=2,$ tehát nem differenciálható.

3.6. Nem differenciálható, mert

$f'_-(1) = \lim\limits_{1-0}\dfrac{x^2-1}{x-1} = 2, \quad f'_+(1) = \lim\limits_{1+0}\dfrac{x-1}{x-1} = 1.$

3.7. $\lim\limits_{0}\dfrac{f(x)-f(0)}{x-0} = \lim\limits_{0}|x| = 0.$

Így f a 0 pontban differenciálható és $f'(0)=0.$

3.8. $f'(0)=+\infty.$

3.9. $f'_+(0) = \lim\limits_{0+0}\dfrac{1}{\sqrt[3]{x}} = +\infty, \quad f'_-(0) = \lim\limits_{0-0}\dfrac{1}{\sqrt[3]{x}} = -\infty,$

tehát a függvény a 0 pontban nem differenciálható.

3.10. $f': f'(x) = 2, \quad x\in\mathrm{R}.$

3.11. $f': f'(x) = 6x^5, \quad x \in \mathrm{R}.$

3.12. $f': f'(x) = -\frac{2}{x^3}, \quad x \in \mathrm{R} \setminus \{0\}.$

3.13. $f': f'(x) = 2 - \frac{1}{x^2}, \quad x \in \mathrm{R} \setminus \{0\}$

3.14. $f': f'(x) = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}, \quad x \in \mathrm{R}^+.$

3.15. $f': f'(x) = \frac{4}{5} x^{-\frac{1}{5}} = \frac{4}{5\sqrt[5]{x}}, \quad x \in \mathrm{R}^+.$

3.16. $f: f(x) = x^{\frac{14}{3}}, \quad x \in \mathrm{R}^+,$

$f': f'(x) = \frac{14}{3} x^{\frac{11}{3}}, \quad x \in \mathrm{R}^+.$

3.17. $f: f(x) = x^{\frac{5}{6}}, \quad x \in \mathrm{R}^+,$

$f': f'(x) = \frac{5}{6} x^{-\frac{1}{6}}, \quad x \in \mathrm{R}^+.$

3.18. $f': f'(x) = 20x^4 - 6x, \quad x \in \mathrm{R}.$

3.19. $f(x) = x^{-\frac{4}{3}} + 2x^3 - x^{-\frac{3}{2}}.$

$f'(x) = -\frac{4}{3} x^{-\frac{7}{3}} + 6x^2 + \frac{3}{2} x^{-\frac{5}{2}}, \quad x \in \mathrm{R}^+.$

3.20. $f': f'(x) = \frac{2}{(x-2)^2}, \quad x \in \mathrm{R} \setminus \{2\}.$

3.21. $f'\colon f'(x) = \dfrac{-(2x+1)}{\left(x^2+x-6\right)^2}, \quad x \in \mathrm{R} \setminus \{-3,\ 2\}.$

3.22. $f'\colon f'(x) = \dfrac{x}{\left(x^2+1\right)^2}, \quad x \in \mathrm{R}.$

3.23. $f'\colon f'(x) = \dfrac{6x}{\left(2x^2+1\right)^2}, \quad x \in \mathrm{R}.$

3.24. $f'\colon f'(x) = \dfrac{-18x-10}{x^{11}}, \quad x \in \mathrm{R} \setminus \{0\}.$

3.25. $f'\colon f'(x) = 2^x \ln 2 - \dfrac{\ln 2}{2^x}, \quad x \in \mathrm{R}.$

3.26. $f'\colon f'(x) = e^{-x^2} \cdot (-2x) + e^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}}, \quad x \in \mathrm{R}^+.$

3.27. $f'(x) = \dfrac{5}{1+3x^2} \cdot 6x = \dfrac{30x}{1+3x^2}, \quad x \in \mathrm{R}^+.$

3.28. $f'\colon f'(x) = \ln x, \quad x \in \mathrm{R}^+.$

3.29. $f'\colon f'(x) = \dfrac{1-2\ln x}{x^3}, \quad x \in \mathrm{R}^+.$

3.30. $f'\colon f'(x) = \dfrac{\ln x - 1}{\ln^2 x}, \quad x \in \mathrm{R}^+ \setminus \{1\}.$

3.31. $f'\colon f'(x) = \dfrac{1}{x \ln a}, \quad x \in \mathrm{R}^+.$

3.32. $f'\colon f'(x) = \dfrac{1}{x \ln 10} = \dfrac{0{,}4343}{x}, \quad x \in \mathrm{R}^+.$

3.33. $f'\colon f'(x) = \dfrac{-1}{x \ln^2 x}, \quad x \in \mathrm{R}^+ \setminus \{1\}.$

3.34. $f': f'(x) = \dfrac{\ln x}{2\sqrt{x}} + \dfrac{\sqrt{x}}{x}, \quad x \in \mathrm{R}^+.$

3.35. $f': f'(x) = e^x \ln x + \dfrac{e^x}{x}, \quad x \in \mathrm{R}^+.$

3.36. $f': f'(x) = \dfrac{e^x(x\ln x - 1)}{x\ln^2 x}, \quad x \in \mathrm{R}^+ \setminus \{1\}.$

3.37. $f': f'(x) = 2(4x^2 + 5x + 6) + (2x + 3)(8x + 5) = 24x^2 + 44x + 27,$
$x \in \mathrm{R}.$

3.38. $f': f'(x) = 12x^2 - 4x + 6 - \dfrac{2}{\sqrt{x}}, \quad x \in \mathrm{R}^+.$

3.39. $f': f'(x) = \dfrac{-(2x - 4)}{(x^2 - 4x + 4)^2}, \quad x \in \mathrm{R} \setminus \{2\}.$

3.40. $f': f'(x) = \dfrac{3(1 + x^4) - 3x \cdot 4x^3}{(1 + x^4)^2} = \dfrac{3 + 3x^4 - 12x^4}{(1 + x^4)^2} = \dfrac{3 - 9x^4}{(1 + x^4)^2}, \ x \in \mathrm{R}.$

3.41. $f': f'(x) = \dfrac{8(1 + x^2) - 8x \cdot 2x}{(1 + x^2)^2} = \dfrac{8 + 8x^2 - 16x^2}{(1 + x^2)^2} =$

$= \dfrac{8 - 8x^2}{(1 + x^2)^2}, \quad x \in \mathrm{R}.$

3.42. $f': f'(x) = \dfrac{-4}{(2x + 1)^2}, \quad x \in \mathrm{R} \setminus \left\{-\dfrac{1}{2}\right\}.$

3.43. $f': f'(x) = \dfrac{-5x + 10}{(x + 2)^3}, \quad x \in \mathrm{R}^+.$

3.44. $f'\colon f'(x) = -\dfrac{2x+2}{(x-2)^4}, \quad x \in \mathrm{R} \setminus \{2\}.$

3.45. $f'\colon f'(x) = \dfrac{6x^4 - 32x^3 - 51x^2 - 30x - 1}{\left(3x^2 - 4x - 7\right)^2}, \quad x \in \mathrm{R} \setminus \left\{\dfrac{7}{3};\ -1\right\}.$

3.46. $f\colon f(x) = \dfrac{3^2}{3^x},$

$f'\colon f'(x) = \dfrac{-9 \cdot 3^x \cdot \ln 3}{3^{2x}} = \dfrac{-9 \cdot \ln 3}{3^x}, \quad x \in \mathrm{R}.$

3.47. $f'\colon f'(x) = 6\left(2^{5x} + 3x\right)^5 \left(5 \cdot 2^{5x} \cdot \ln 2 + 3\right).$

3.48. $f'\colon f'(x) = \dfrac{\dfrac{1}{2\sqrt{x}}\left(\sqrt[3]{x} + 1\right) - \left(\sqrt{x} + 8\right)\dfrac{1}{3}x^{-\frac{2}{3}}}{\left(\sqrt[3]{x} + 1\right)^2}, \quad x \in \mathrm{R}^+.$

3.49. $f'\colon f'(x) = -\dfrac{2}{3\sqrt[3]{1-x}}, \quad x \in \mathrm{R} \setminus \{1\}.$

3.50. $f'\colon f'(x) = -\dfrac{1}{(1+x)\sqrt{1-x^2}}, \quad x \in]-1,1[.$

3.51. $f'\colon f'(x) = 1 - \left(\dfrac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2, \quad x \in \mathrm{R} \setminus \{0\}.$

3.52. $f'\colon f'(x) = e^{x^2+5x+1}\left[(2x+5)\ln x + \dfrac{1}{x}\right], \quad x \in \mathrm{R}^+.$

3.53. $f'\colon f'(x) = -\dfrac{3x^2\, e^{\frac{-x^3}{4}}}{4\sqrt{2p}}, \quad x \in \mathrm{R}.$

3.54. $f'\colon f'(x) = e^x \ln x + \dfrac{e^x}{x} + 6x\left(x^2-4\right)^2 + \dfrac{1}{2\sqrt{x}}, \quad x \in \mathrm{R}^+.$

3.55. $f'\colon f'(x) = (-2x+2)e^{-x^2+2x+1}\ln x + \dfrac{e^{-x^2+2x+1}}{x}, \quad x \in \mathrm{R}^+.$

3.56. $f'\colon f'(x) = \sqrt[4]{6x^2+e^{2x}} + x \cdot \dfrac{1}{4}\left(6x^2+e^{2x}\right)^{-\frac{3}{4}} \cdot \left(12x + e^{2x}\cdot 2\right), \quad x \in \mathrm{R}.$

3.57. $f'\colon f'(x) = \left(x^{\frac{5}{6}} - x^{-\frac{1}{6}}\right)' = \dfrac{5}{6}\cdot x^{-\frac{1}{6}} + \dfrac{1}{6}\, x^{-\frac{7}{6}}, \quad x \in \mathrm{R}^+.$

3.58. $f'\colon f'(x) = 3x^2 \cdot \ln x + \dfrac{x^3}{x} = x^2(3\ln x + 1), \quad x \in \mathrm{R}^+.$

3.59. $f'\colon f'(x) = 4x^3 \ln 3x + x^4 \cdot \dfrac{1}{3x}\cdot 3 = 4x^3 \ln 3x + x^3, \quad x \in \mathrm{R}^+.$

3.60. $f'\colon f'(x) = \dfrac{\left(3^x \cdot \ln 3 + 6x^2\right)\left(3x+\sqrt{x}\right) - \left(3^x + 2x^3\right)\left(3 + \dfrac{1}{2\sqrt{x}}\right)}{\left(3x+\sqrt{x}\right)^2}, \quad x \in \mathrm{R}^+.$

3.61. $f'\colon f'(x) = \ln 5 \cdot 5^{\sqrt{\frac{1-x}{1+x}}} \cdot \dfrac{1}{2}\left(\dfrac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \dfrac{-1(1+x)-(1-x)}{(1+x)^2} =$

$= \ln 5 \cdot 5^{\sqrt{\frac{1-x}{1+x}}} \cdot \dfrac{1}{2}\left(\dfrac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \left(-\dfrac{2}{(1+x)^2}\right), \quad x \in]-1,1[.$

3.62. $f'\colon f'(x) = 3x^2 \cdot e^{4x} + 4x^3 \cdot e^{4x} + 2^{1-x^2}\ln 2(-2x), \quad x \in \mathrm{R}.$

3.63. $f'\colon f'(x) = \dfrac{2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}, \quad x \in \mathrm{R}^+.$

3.64. $f'\colon f'(x) = \dfrac{-3x}{\left(1+x^2\right)^2\sqrt{1+x^2}}, \quad x \in \mathrm{R}.$

3.65. $f'\colon f'(x) = \dfrac{e^x}{1+e^x}, \quad x \in \mathrm{R}.$

3.66. $f'\colon f'(x) = \dfrac{1}{x\ln x}, \quad x \in]1,\infty[.$

3.67. $f'\colon f'(x) = -\dfrac{3}{x(4x-3)}, \quad x \in \mathrm{R}^-.$

3.68. $f'\colon f'(x) = \dfrac{2e^x}{1-e^{2x}}, \quad x \in \mathrm{R}^-.$

3.69. $f'\colon f'(x) = \dfrac{1}{1-x^2}, \quad x \in]-1;\ 1[.$

3.70. $f'\colon f'(x) = \dfrac{1}{x-\sqrt{1-x^2}}\left(1-\dfrac{1}{2\sqrt{1-x^2}}\,(-2x)\right), \quad x \in \left]\dfrac{\sqrt{2}}{2},\ 1\right[.$

3.71. $f'\colon f'(x) = \dfrac{1}{1+e^{2x}}, \quad x \in \mathrm{R}^+.$

3.72. $f'\colon f'(x) = 4e^{4x}\ln x + \dfrac{e^{4x}}{x} + e^{2x^4+3x}\left(8x^3+3\right), \quad x \in \mathrm{R}^+.$

3.73. $f'\colon f'(x) = 2xe^{x^2-5}\sqrt{x+3} + \dfrac{e^{x^2-5}}{2\sqrt{x+3}}, \quad x \in]-3,\infty[.$

3.74. $f'\colon f'(x) = \dfrac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-\frac{1}{2}} \cdot \left(1+\dfrac{1}{2}\left(x+\sqrt{x}\right)^{-\frac{1}{2}}\right)\left(1+\dfrac{1}{2\sqrt{x}}\right) =$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\right) \cdot \left(1+\frac{1}{2\sqrt{x}}\right), \quad x \in \mathrm{R}^+.$$

3.75. $f': f'(x) = \dfrac{1}{\left(1+3x^2\right)^4} \cdot 4\left(1+3x^2\right)^3 \cdot 6x, \quad x \in \mathrm{R}^+.$

3.76. $f': f'(x) = \dfrac{1}{x^2+3x} \cdot (2x+3) + e^{-2x^5}\left(-10x^4\right), \quad x \in \mathrm{R}^+.$

3.77. $\ln f(x) = x \ln x$

$\left[\ln f(x)\right]' = \ln x + 1 \Rightarrow f'(x) = x^x(\ln x + 1), \quad x \in \mathrm{R}^+ \setminus \{1\}.$

3.78. $f': f'(x) = (3+2x)^x\left(\ln(3+2x) + \dfrac{2x}{3+2x}\right), \quad x \in \mathrm{R}^+.$

3.79. $f': f'(x) = \left(1+\dfrac{1}{x}\right)^x\left[\ln\left(1+\dfrac{1}{x}\right) - \dfrac{1}{x+1}\right], \quad x \in \mathrm{R}^+.$

3.80. $\ln f(x) = \dfrac{1}{x} \cdot \ln x$

$\dfrac{1}{f(x)} \cdot f'(x) = -\dfrac{1}{x^2}\ln x + \dfrac{1}{x^2}$

$f': f'(x) = \sqrt[x]{x}\left(\dfrac{1-\ln x}{x^2}\right), \quad x \in \mathrm{R}^+.$

3.81. $\ln f(x) = x^2 \cdot \ln\left(3+x^2\right)$

$\dfrac{1}{f(x)} \cdot f'(x) = 2x \cdot \ln\left(3+x^2\right) + \dfrac{x^2 \cdot 2x}{3+x^2}$

$f': f'(x) = \left(3+x^2\right)^{x^2}\left[2x \cdot \ln\left(3+x^2\right) + \dfrac{2x^3}{3+x^2}\right], \quad x \in \mathrm{R}^+.$

3.82. $\ln f(x) = \dfrac{1}{x} \cdot \ln(4x+5)$

$\dfrac{1}{f(x)} \cdot f'(x) = -\dfrac{1}{x^2} \cdot \ln(4x+5) + \dfrac{4}{x(4x+5)}$

$f': f'(x) = \sqrt[x]{4x+5}\left[\dfrac{4}{x(4x+5)} - \dfrac{1}{x^2}\ln(4x+5)\right], \quad x \in \mathrm{R}^+.$

3.83. $\ln f(x) = \left(3x^2+5x-1\right)\cdot \ln x$

$$\frac{1}{f(x)}\cdot f'(x) = (6x+5)\cdot \ln x + \frac{\left(3x^2+5x-1\right)}{x}$$

$$f': f'(x) = x^{3x^2+5x-1}\left[(6x+5)\ln x + \frac{3x^2+5x-1}{x}\right], \quad x\in \mathrm{R}^+\setminus\{1\}.$$

3.84. $\ln f(x) = x\cdot \ln\ln x$

$$\frac{1}{f(x)}\cdot f'(x) = \ln\ln x + \frac{x}{\ln x}\cdot\frac{1}{x}$$

$$f': f'(x) = (\ln x)^x\left[\ln\ln x + \frac{1}{\ln x}\right], \quad x\in \mathrm{R}^+.$$

3.85. $\ln f(x) = (\ln x)\cdot(\ln\ln x)$

$$\frac{1}{f(x)}\cdot f'(x) = \frac{1}{x}\left[\ln\ln x+1\right]$$

$$f': f'(x) = (\ln x)^{\ln x}\ \frac{1}{x}\left[\ln\ln x+1\right], \quad x\in \mathrm{R}^+.$$

3.86. $f': f'(x) = \frac{1}{x\ln x}, \quad x\in \mathrm{R}^+.$

3.87. $\ln f(x) = 5x^4\cdot \ln\ln 3x$

$$\frac{1}{f(x)}\cdot f'(x) = 20x^3\ln\ln 3x + \frac{5x^4\cdot 3}{\ln 3x}$$

$$f': f'(x) = (\ln 3x)^{5x^4}\left(20x^3\ln\ln 3x + \frac{15x^4}{\ln 3x}\right), \quad x\in \mathrm{R}^+.$$

3.88. $\ln f(x) = \sqrt{9-x^2}\cdot \ln x$

$$\frac{1}{f(x)}\cdot f'(x) = \frac{1}{2\sqrt{9-x^2}}(-2x)\ln x + \frac{\sqrt{9-x^2}}{x}$$

$$f': f'(x) = x^{\sqrt{9-x^2}}\left[\frac{-x\ln x}{\sqrt{9-x^2}} + \frac{\sqrt{9-x^2}}{x}\right], \quad x\in]2,3[.$$

3.89. $f': f'(x) = \dfrac{x}{x^4 - 1}, \quad x \in \mathrm{R} \setminus [-1;1].$

3.90. $f': f'(x) = \dfrac{4x + 2}{x^2 + x + 1}, \quad x \in \mathrm{R}.$

3.91. $f': f'(x) = 4x^3 \ln x + x^3, \quad x \in \mathrm{R}^+$
$f': f'(1) = 1.$

3.92. $f': f'(x) = \dfrac{1}{2\sqrt{x}} \ln x + \dfrac{1}{\sqrt{x}}, \quad x \in \mathrm{R}^+$
$f'(e) = \dfrac{1}{2\sqrt{e}} + \dfrac{1}{\sqrt{e}} = \dfrac{3}{2\sqrt{e}}.$

3.93. $f': f'(x) = e^x\left(x^2 + 3x\right) + e^x(2x + 3), \quad x \in \mathrm{R}, \quad f'(1) = 9e.$

3.94. A g függvény csak a 0 pontban differenciálható.
Itt $g'(0) = 0$.

3.95. Lehetséges.

3.96. $f': f'(x) = 6x^2, \quad x \in \mathrm{R}$. A párhuzamosság feltétele alapján az érintő iránytangense: 6. A $6x^2 = 6$ egyenletet megoldva két érintési pontot kapunk: $P_1(1, 5)$ és $P_2(-1, 1)$.

3.97. $f': f'(x) = -\dfrac{8}{x^2}, \quad x \in \mathrm{R} \setminus \{0\}$. Két pontot kapunk: $P_1(2\sqrt{6}; \dfrac{2\sqrt{6}}{3})$ és $P_2(-2\sqrt{6}; -\dfrac{2\sqrt{6}}{3})$.

3.98. $f': f'(x) = -\dfrac{3x^2 + 4x + 3}{\left(x^2 - 1\right)^2}; \quad x \in \mathrm{R} \setminus \{-1,1\}, \quad f'(2) = -\dfrac{23}{9}.$

3.99. P (1; 6)
$f': f'(x) = 2x + 6$
$f'(1) = 8 = m$

$y - 6 = 8(x - 1)$

$y = 8x - 2$.

3.100. A függvény az x tengelyt az $x = -1$ és $x = 1$ helyeken metszi.
A keresett érintők: $f_1(x) = 2x - 2$,
$f_2(x) = 2x + 2$.

3.101. $f(x) = -12x + 9$.

3.102. f': $f'(x) = 2x + 1$ $\quad$ E(a,b).

$y - b = (2a + 1)(x - a)$

$b = a^2 + a - 6$

$4 - (a^2 + a - 6) = (2a + 1)(3 - a)$

$4 - a^2 - a + 6 = 6a - 2a^2 + 3 - a$

$a^2 - 6a + 7 = 0$

$a_{1,2} = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$, és visszahelyettesítünk.

3.103. a.) $f'(x) = -\frac{1}{x+2}$, $\quad f'(-1) = -1$, $\quad m = -1$.

b.) $\alpha = 135^o$.

c.) $f(-1) = 0$, $\quad$ P(-1;0), $\quad y = -x - 1$ az érintő egyenlete.

3.104. a.) $f'(x) = -\frac{3}{2}(1 - x)^2$, $\quad f'(-1) = -6$, $\quad m = -6$.

b.) $\alpha = 99{,}5^o$.

c.) $f(-1) = 4$, $\quad$ P(-1;4), $\quad y = -6x - 2$ az érintő egyenlete.

3.105. f''': $f'''(x) = 240x^3 + 120x$, $\quad x \in \mathrm{R}$.

3.106. f''': $f'''(x) = \frac{15}{64x^2 \sqrt[4]{x}}$, $\quad x \in \mathrm{R}^+$.

3.107. f''': $f'''(x) = 2x(12 \ln x + 7) + x^2 \frac{12}{x} = 2x(12 \ln x + 13)$, $\quad x \in \mathrm{R}^+$.

3.108. f'': $f''(x) = 120(2x + 10)^4$, $\quad x \in \mathrm{R}$.

3.109. $f''': f'''(x) = 60x^2 - 12, \quad x \in \mathrm{R}.$

3.110. $f''': f'''(x) = \frac{24}{x^3}, \quad x \in \mathrm{R}^+.$

3.111. $f''': f'''(x) = e^x\left(x^4 + 12x^3 + 36x^2 + 24x\right), \quad x \in \mathrm{R}.$

3.112. $f'''(x) = (5\ln 2)^3\ 2^{5x}.$

3.113. $f'''(x) = 6.$

3.114. $f''(x) = e^x\sqrt{x} + \frac{e^x}{2\sqrt{x}} + \frac{2e^x\ \sqrt{x} - e^x \cdot e^{-\frac{1}{2}}}{4x}, \quad x \in \mathrm{R}^+.$

3.115. $f''(x) = \frac{-3}{x^2}, \quad x \in \mathrm{R}^+.$

3.116. $f'''(x) = \frac{e^x - e^{-x}}{2}, \quad x \in \mathrm{R}.$

3.117. $f'_x(2,0) = 1, \quad f'_y(2,0) = 0.$

3.118. $f'_x(1,2) = \frac{4}{\sqrt{36}} = \frac{2}{3}.$

3.119. $f'_x: f'_x(x,y) = 3x^2 + y, \quad (x,y) \in \mathrm{R}^2.$

3.120. $f'_x: f'_x(x,y) = e^y - 2xy, \quad (x,y) \in \mathrm{R}^2.$

3.121. $f'_x: f'_x(x,y) = 2^x \ln 2 + e^x y^2 + \frac{y}{2\sqrt{x}},$

$$D_{f'_x} = \left\{(x,y) \in \mathrm{R}^2 \,|\, x > 0, \quad y \in \mathrm{R}\right\}.$$

3.122. $f'_x: f'_x(x,y) = 2xy^3 + \frac{y}{x}, \quad D_{f'_x} = \left\{(x,y) \in \mathrm{R}^2 \,|\, x > 0, \quad y \in \mathrm{R}\right\}.$

$f'_y: f'_y(x,y) = 3x^2y^2 + \ln x, \quad D_{f'_y} = \left\{(x,y) \in \mathrm{R}^2 \,|\, x > 0, \quad y \in \mathrm{R}\right\}.$

3.123. $f'_x\colon f'_x(x,y) = 2xe^{x^2+y^2}, \quad (x,y)\in \mathrm{R}^2.$

$f'_y\colon f'_y(x,y) = 2ye^{x^2+y^2}, \quad (x,y)\in \mathrm{R}^2.$

3.124. $f'_x\colon f'_x(x,y) = y^2e^{xy^2}, \quad (x,y)\in \mathrm{R}^2.$

$f'_y\colon f'_y(x,y) = 2xye^{xy^2}, \quad (x,y)\in \mathrm{R}^2.$

3.125. $f'_x\colon f'_x(x,y) = \dfrac{2x}{x^2+y^2}, \quad D_{f'_x} = D_f.$

$f'_y\colon f'_y(x,y) = \dfrac{2y}{x^2+y^2}, \quad D_{f'_y} = D_f.$

3.126. $f'_x\colon f'_x(x,y) = e^{x+y}\ln\left(x^2+y^2\right) + \dfrac{e^{x+y}}{x^2+y^2}\cdot 2x, \quad D_{f'_x} = D_f$

$f'_y\colon f'_y(x,y) = e^{x+y}\ln\left(x^2+y^2\right) + \dfrac{e^{x+y}}{x^2+y^2}\cdot 2y, \quad D_{f'_y} = D_f.$

3.127. $f'_x\colon f'_x(x,y) = \dfrac{1}{2}\sqrt{\dfrac{y}{x}} + y^{-\frac{3}{2}}, \quad D_{f'_x} = D_f.$

$f'_y\colon f'_y(x,y) = \dfrac{1}{2}\sqrt{\dfrac{x}{y}} - \dfrac{3}{2}xy^{\frac{-5}{2}}, \quad D_{f'_y} = D_f.$

3.128. $f'_x\colon f'_x(x,y) = 6x^2+6xy, \quad (x,y)\in \mathrm{R}^2.$

$f'_y\colon f'_y(x,y) = 3x^2-4y, \quad (x,y)\in \mathrm{R}^2.$

$f''_{xx}\colon f''_{xx}(x,y) = 12x+6y, \quad (x,y)\in \mathrm{R}^2.$

$f''_{yy}\colon f''_{yy}(x,y) = -4, \quad (x,y)\in \mathrm{R}^2.$

$f''_{xy}\colon f''_{xy}(x,y) = 6x, \quad (x,y)\in \mathrm{R}^2.$

$f''_{yx}\colon f''_{yx}(x,y) = 6x, \quad (x,y)\in \mathrm{R}^2.$

3.129. $f'_x\colon f'_x(x,y) = 2x-3y+1, \quad (x,y)\in \mathrm{R}^2.$

$f'_y\colon f'_y(x,y) = -3x+4y-4, \quad (x,y)\in \mathrm{R}^2.$

$f''_{xx}: f''_{xx}(x,y) = 2, \quad (x,y) \in \mathrm{R}^2.$

$f''_{yy}: f''_{yy}(x,y) = 4, \quad (x,y) \in \mathrm{R}^2.$

$f''_{xy}: f''_{xy}(x,y) = -3, \quad (x,y) \in \mathrm{R}^2.$

$f''_{yx}: f''_{yx}(x,y) = -3, \quad (x,y) \in \mathrm{R}^2.$

3.130. $f'_x: f'_x(x,y) = e^{-x^2}\left(1 - 2x^2 + 10xy - 16x\right).$

$f'_y: f'_y(x,y) = -5 \cdot e^{-x^2}.$

$f''_{xx}: f''_{xx}(x,y) =$

$= e^{-x^2}(-2x) + 4x \cdot e^{-x^2} + 2x^2 e^{-x^2}(-2x) + 10y\left(e^{-x^2} + xe^{-x^2}(-2x)\right) -$

$-16\left(e^{-x^2} + xe^{-x^2}(-2x)\right).$

$f''_{yy}: f''_{yy}(x,y) = 0.$

$f''_{xy}: f''_{xy}(x,y) = f''_{yx}: f''_{yx}(x,y) = 10xe^{-x^2}. \quad (x,y) \in \mathrm{R}^2.$

3.131. $f'_x: f'_x(x,y) = -2 \cdot e^{-y^2}.$

$f'_y: f'_y(x,y) = -6ye^{-y^2} + 4xye^{-y^2} + e^{-y^2} - 2y^2 \cdot e^{-y^2}.$

$f''_{xy}: f''_{xy}(x,y) = f''_{yx}: f''_{yx}(x,y) = 4ye^{-y^2}.$

$f''_{xx}: f''_{xx}(x,y) = 0.$

$f''_{yy}: f''_{yy}(x,y) =$

$= -6e^{-y^2} - 6ye^{-y^2}(-2y) + 4x\left(e^{-y^2} + ye^{-y^2}(-2y)\right) + e^{-y^2}(-2y) -$

$-2\left(2ye^{-y^2} + y^2 \cdot e^{-y^2}(-2y)\right). \quad (x,y) \in \mathrm{R}^2.$

3.132. $f'_x: f'_x(x,y) = 2x + e^y + 3x^2y^2.$

$f'_y: f'_y(x,y) = x \cdot e^y + 2x^3y.$

$f''_{xx}: f''_{xx}(x,y) = 2 + 6xy^2.$

$f''_{yy}: f''_{yy}(x,y) = x \cdot e^y + 2x^3.$

$f''_{xy} = f''_{yx} = e^y + 6x^2 y. \quad (x,y) \in \mathrm{R}^2.$

3.133. $f'_x: f'_x(x,y) = 1 - e^y.$

$f'_y: f'_y(x,y) = 1 - x \cdot e^y.$

$f''_{xx}: f''_{xx}(x,y) = 0.$

$f''_{yy}: f''_{yy}(x,y) = -x \cdot e^y.$

$f''_{xy} = f''_{yx} = -e^y. \quad (x,y) \in \mathrm{R}^2.$

3.134. $f'_x: f'_x(x,y) = \frac{1}{y} \cdot e^{\frac{x}{y}}, \qquad D_{f'_x} = D_f.$

$f'_y: f'_y(x,y) = -\frac{x}{y^2} \cdot e^{\frac{x}{y}}, \quad D_{f'_y} = D_f.$

$f''_{xx}: f''_{xx}(x,y) = \frac{1}{y^2} e^{\frac{x}{y}}.$

$f''_{yy}: f''_{yy}(x,y) = e^{\frac{x}{y}} \left(\frac{2x}{y^3} + \frac{x^2}{y^4} \right).$

$f''_{xy}: f''_{xy}(x,y) = -e^{\frac{x}{y}} \left(\frac{1}{y^2} + \frac{x}{y^3} \right).$