2. FÜGGVÉNYEK HATÁRÉRTÉKE, FOLYTONOSSÁGA

2.1. 4.

2.2. $\frac{1}{3}$.

2.3. 0.

2.4. Csak jobb oldali határérték létezik: 0.

2.5. 2, mert $\frac{x^2-9}{x^2-3x}=\frac{(x-3)(x+3)}{x(x-3)}=\frac{(x-3)}{(x-3)}\ \frac{(x+3)}{x}$.

2.6. $-\frac{1}{4}$, mert $\frac{x+2}{x^2-4}=\frac{x+2}{x+2}\ \frac{1}{x-2}$.

2.7. 0.

2.8. $-\frac{2}{3}$, mert $\frac{x^2+2x-3}{x^2-8x+7}=\frac{(x-1)(x+3)}{(x-1)(x-7)}$.

2.9. 3.

2.10. $\frac{1}{2}$.

2.11. 0.

2.12. $\frac{4}{3}$.

2.13. $\frac{1}{2}$.

2.14. $\frac{1}{2\sqrt{a}}$, mert $\frac{\sqrt{x}-\sqrt{a}}{x-a}=\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x}-\sqrt{a}}\ \frac{1}{\sqrt{x}+\sqrt{a}}$.

2.15. $\frac{1}{4}$, mert $\frac{\sqrt{x+4}-2}{x}=\frac{(x+4)-4}{x(\sqrt{x+4}+2)}=\frac{x}{x}\ \frac{1}{\sqrt{x+4}+2}$.

2.16. $\frac{1}{12}$, mert $\frac{\sqrt[3]{x}-2}{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}=\frac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}$.

2.17. $\lim\limits_{4}\dfrac{\left(x^2-4\right)\left(x^2-1\right)}{\left(x^2-4\right)\left(x^2+2\right)}=\dfrac{5}{6}.$

2.18. $\lim\limits_{-2}\dfrac{(x+2)(x-2)\left(x^2+6\right)}{(x-2)(x+2)}=\lim\limits_{-2}\left(x^2+6\right)=10.$

2.19. $\lim\limits_{1}\dfrac{(x-1)\sqrt{5x^2+4}}{x^2+x-2}=\lim\limits_{1}\dfrac{(x-1)\sqrt{5x^2+4}}{(x-1)(x+2)}=1.$

2.20. $\lim\limits_{-\sqrt{5}}\dfrac{x^4-2x^2-15}{x^4-6x^2+5}=\lim\limits_{-\sqrt{5}}\dfrac{\left(x^2-5\right)\left(x^2+3\right)}{\left(x^2-5\right)\left(x^2-1\right)}=2.$

2.21. $\lim\limits_{2}\dfrac{(x-2)(x+2)\left(x^2+4\right)}{(x-2)(x-3)}=-32.$

2.22. $\lim\limits_{\sqrt{3}}\dfrac{x^2-3\sqrt{3}x+6}{x^2-3}=\lim\limits_{\sqrt{3}}\dfrac{\left(x-\sqrt{3}\right)\left(x-2\sqrt{3}\right)}{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}=-\dfrac{1}{2}.$

2.23. $\lim\limits_{1}\dfrac{3(x+1)-2\left(x^2+x+1\right)}{(x+1)(x-1)\left(x^2+x+1\right)}=\lim\limits_{1}\dfrac{3x+3-2x^2-2x-2}{(x+1)(x-1)\left(x^2+x+1\right)}=$

$=\lim\limits_{1}\dfrac{x-2x^2+1}{(x+1)(x-1)\left(x^2+x+1\right)}=\lim\limits_{1}\dfrac{(1-x)(2x+1)}{(x+1)(x-1)\left(x^2+x+1\right)}=$

$=\lim\limits_{1}\dfrac{-(2x+1)}{(x+1)\left(x^2+x+1\right)}=\dfrac{-3}{6}=-\dfrac{1}{2}.$

2.24. $\lim\limits_{1}\dfrac{(x-1)(x+1)\left(\sqrt{2-x}+\sqrt{x}\right)}{2-x-x}=\lim\limits_{1}\dfrac{-(x+1)\left(\sqrt{2-x}+\sqrt{x}\right)}{2}=-2.$

2.25. $\dfrac{x^2-a^2}{x^3-a^3}=\dfrac{(x-a)(x+a)}{(x-a)\left(x^2+ax+a^2\right)}=\dfrac{x+a}{x^2+ax+a^2}$

$\lim\limits_{a}\dfrac{x+a}{x^2+ax+a^2}=\dfrac{2a}{3a^2}=\dfrac{2}{3a}.$

2.26. $\lim\limits_{2}\dfrac{x^2-5x+6}{x^3-2x^2-x+2} = \lim\limits_{2}\dfrac{2x-5}{3x^2-4x-1} = -\dfrac{1}{3}.$

2.27. $\lim\limits_{1}\dfrac{x^6-1}{x^5-1} = \lim\limits_{1}\dfrac{6x^5}{5x^4} = \dfrac{6}{5}.$

2.28. $\lim\limits_{0}\dfrac{e^x-e^{-x}}{x} = \lim\limits_{0}\left(e^x+e^{-x}\right) = 2.$

2.29. -1, mert $\dfrac{x^2(x-1)}{x^2\left(x^2+x+1\right)} = \dfrac{x-1}{x^2+x+1}.$

2.30. $\lim\limits_{5}\dfrac{\left(2-\sqrt{x-1}\right)\left(2+\sqrt{x-1}\right)}{(x-5)(x+5)\left(2+\sqrt{x-1}\right)} = \lim\limits_{5}\dfrac{4+1-x}{(x-5)(x+5)\left(2+\sqrt{x-1}\right)} =$

$= \lim\limits_{5}\dfrac{-1}{(x+5)\left(2+\sqrt{x-1}\right)} = -\dfrac{1}{40}.$

2.31. $\lim\limits_{1}\dfrac{(x-1)(x+5)}{(x-1)(x+7)} = \dfrac{3}{4}.$

2.32. $\lim\limits_{3}\dfrac{(x-3)(x+8)}{(x-3)(x-5)} = -\dfrac{11}{2}.$

2.33. $\dfrac{x^2+2x-15}{x^2-4x+3} = \dfrac{(x-3)(x+5)}{(x-3)(x-1)} = \dfrac{x+5}{x-1}$

$\lim\limits_{3}\dfrac{x^2+2x-15}{x^2-4x+3} = \dfrac{8}{2} = 4.$

2.34. $\dfrac{\sqrt{x+3}-3}{\sqrt{(x+3)^3}-27} = \dfrac{\sqrt{x+3}-3}{\left(\sqrt{x+3}-3\right)\left(\sqrt{(x+3)^2}+3\sqrt{x+3}+9\right)}$

$\lim\limits_{6}\dfrac{\sqrt{x+3}-3}{\sqrt{(x+3)^3}-27} = \dfrac{1}{27}.$

2.35. $\dfrac{\frac{1}{x^2}-1}{x-1}=\dfrac{1-x^2}{x^2(x-1)}=\dfrac{-(x-1)(x+1)}{(x-1)x^2}=-\dfrac{x+1}{x^2}$

$\lim\limits_{1}\dfrac{\frac{1}{x^2}-1}{x-1}=-2.$

2.36. $\dfrac{x^n-1}{x-1}=\dfrac{(x-1)\left(x^{n-1}+x^{n-2}+\text{L}+x^0\right)}{x-1}$

$\lim\limits_{1}\dfrac{x^n-1}{x-1}=n.$

2.37. $\lim\limits_{0}\dfrac{\ln x}{\frac{1}{x}}=\lim\limits_{0}\dfrac{\frac{1}{x}}{-x^{-2}}=-\lim\limits_{0}x=0.$

2.38. $\lim\limits_{0}\dfrac{x+1-e^x}{x\left(1-e^x\right)}=\lim\limits_{0}\dfrac{1-e^x}{1-e^x-xe^x}=\lim\limits_{0}\dfrac{-e^x}{-e^x-e^x-xe^x}=\dfrac{-1}{-2}=\dfrac{1}{2}.$

2.39. $\lim\limits_{2+0}\left(3x+\dfrac{4}{x-2}\right)=+\infty,\ \lim\limits_{2-0}\left(3x+\dfrac{4}{x-2}\right)=-\infty\ .$

2.40. $\lim\limits_{0+0}\dfrac{ln\,x^3}{\frac{1}{x^3}}=\lim\limits_{0+0}\dfrac{\frac{1}{x^3}\cdot 3x^2}{-\frac{3}{x^4}}=\lim\limits_{0+0}\left(-\dfrac{3}{x}\right)\cdot\dfrac{x^4}{3}=\lim\limits_{0+0}\left(-x^3\right)=0.$

2.41. $\lim\limits_{0+0}\dfrac{\ln 6x}{\frac{1}{x^2}}=\lim\limits_{0+0}\dfrac{\frac{6}{6x}}{-\frac{2}{x^3}}=\lim\limits_{0+0}\dfrac{1}{x}\cdot\dfrac{x^3}{-2}=\lim\limits_{0+0}\left(-\dfrac{x^2}{2}\right)=0.$

2.42. $\lim\limits_{2+0}\dfrac{x+7}{x^2-5x+6}=-\infty,\ \lim\limits_{2-0}\dfrac{x+7}{x^2-5x+6}=+\infty.$

2.43. $\lim\limits_{1+0}\dfrac{x}{x^2-1}=+\infty,\ \lim\limits_{1-0}\dfrac{x}{x^2-1}=-\infty.$

2.44. $\lim\limits_{1+0}e^{\frac{5}{1-x}}=0,\ \lim\limits_{1-0}e^{\frac{5}{1-x}}=+\infty.$

2.45. $\lim\limits_{2+0}\dfrac{x-2}{|x-2|}=1,\ \lim\limits_{2-0}\dfrac{x-2}{|x-2|}=-1.$

2.46. $\lim\limits_{3+0}\dfrac{7x+1}{x-3}=+\infty,\ \lim\limits_{3-0}\dfrac{7x+1}{x-3}=-\infty.$

2.47. $\lim\limits_{1+0}3^{\frac{1}{x-1}}=+\infty,\ \lim\limits_{1-0}3^{\frac{1}{x-1}}=0.$

2.48. $\lim\limits_{0+0}\dfrac{x-5}{x^2}=+\infty,\ \lim\limits_{0-0}\dfrac{x-5}{x^2}=-\infty.$

2.49. $\lim\limits_{-6+0}\dfrac{x}{x+6}=-\infty,\ \lim\limits_{-6-0}\dfrac{x}{x+6}=+\infty.$

2.50. $\lim\limits_{0+0}\dfrac{|x|}{x}=1,\ \lim\limits_{0-0}\dfrac{|x|}{x}=-1.$

2.51. $\lim\limits_{0+0}\dfrac{x^2+1}{x(x+1)}=+\infty,\ \lim\limits_{0-0}\dfrac{x^2+1}{x(x+1)}=-\infty,$

$\lim\limits_{1+0}\dfrac{x^2+1}{x(x+1)}=1,\ \lim\limits_{1-0}\dfrac{x^2+1}{x(x+1)}=1,$

$\lim\limits_{-1+0}\dfrac{x^2+1}{x(x+1)}=-\infty,\ \lim\limits_{-1-0}\dfrac{x^2+1}{x(x+1)}=+\infty.$

2.52. $\dfrac{1}{3}.$

2.53. $\dfrac{2}{3}$, mert $\dfrac{2x+1}{3x-5}=\dfrac{x\left(2+\frac{1}{x}\right)}{x\left(3-\frac{5}{x}\right)}.$

2.54. 0.

2.55. 2, mert $\dfrac{10x^2+x+1}{5x^2-x+2}=\dfrac{x^2\left(10+\frac{1}{x}+\frac{1}{x^2}\right)}{x^2\left(5-\frac{1}{x}+\frac{2}{x^2}\right)}.$

2.56. $-\infty$, mert $\dfrac{3x - x^2}{x\left(1+\frac{1}{x}\right)} = \dfrac{3-x}{1+\frac{1}{x}}$.

2.57. 0, mert $\dfrac{2x-1}{3x^2+x+2} = \dfrac{x^2\left(\frac{2}{x}-\frac{1}{x^2}\right)}{x^2\left(3+\frac{1}{x}+\frac{2}{x^2}\right)}$.

2.58. $+\infty$.

2.59. 0.

2.60. 1, mert $\dfrac{\sqrt{x^2+1}}{x+1} = \dfrac{x\sqrt{1+\frac{1}{x^2}}}{x\left(1+\frac{1}{x}\right)}$.

2.61. $\lim\limits_{\infty} \dfrac{\sqrt[4]{\frac{1}{x}+\frac{1}{x^3}}-\sqrt{\frac{1}{x}}}{1+\frac{2}{x}+\sqrt{\frac{1}{x}+\frac{1}{x^2}}} = 0.$

2.62. $\lim\limits_{\infty} \dfrac{\left(\sqrt{x}+\sqrt{x+1}\right)x}{-1} = -\infty.$

2.63. $\lim\limits_{\infty} \dfrac{x}{3^x} = \lim\limits_{\infty} \dfrac{1}{3^x \cdot \ln 3} = 0.$

2.64. $\lim\limits_{\infty} \dfrac{e^x}{3} = \infty.$

2.65. $\lim\limits_{\infty} \dfrac{3}{\frac{5}{5x}} = \lim\left(3 \cdot x\right) = \infty.$

2.66. $\lim\limits_{\infty} \dfrac{1}{x} = 0.$

2.67. $\lim\limits_{\infty}\left(\dfrac{x^3-1}{x^3}\right)^{-\frac{x^3}{2}} = \lim\limits_{\infty}\left[\left(1-\dfrac{1}{x^3}\right)^{x^3}\right]^{\frac{-1}{2}} = \left(e^{-1}\right)^{\frac{-1}{2}} = \sqrt{e}.$

2.68. $\lim\limits_{\infty}\left(\frac{2x+3}{2x+1}\right)^{x}\cdot\left(\frac{2x+3}{2x+1}\right)^{2}=\lim\limits_{\infty}\frac{\left(1+\frac{\frac{3}{2}}{x}\right)^{x}}{\left(1+\frac{\frac{1}{2}}{x}\right)^{x}}=e^{\frac{3}{2}-\frac{1}{2}}=e.$

2.69. $\lim\limits_{\infty}\left[\frac{\left(1+\frac{3}{x}\right)^{x}}{\left(1-\frac{2}{x}\right)^{x}}\right]^{\frac{x+3}{x-2}}=\mathrm{e}^{10}.$

2.70. 0, mert $\sqrt{x+3}-\sqrt{x}=\frac{\left(\sqrt{x+3}+\sqrt{x}\right)\left(\sqrt{x+3}-\sqrt{x}\right)}{\sqrt{x+3}+\sqrt{x}}=\frac{x+3-x}{\sqrt{x+3}+\sqrt{x}}.$

2.71. e^2, mert $\left(\frac{x+1}{x}\right)^{2x}=\left[\left(1+\frac{1}{x}\right)^{x}\right]^{2}.$

2.72. Folytonos.

2.73. Folytonos.

2.74. Nem folytonos.

2.75. Folytonos.

2.76. Folytonos.

2.77. Nem folytonos.

2.78. Folytonos.

2.79. Nem folytonos.

2.80. g: $g(x)=x-3$.

2.81. Nincs megoldás.

2.82. x = 1,222.

2.83. x = 2,208.